Large center vortices and confinement in 3D $\mathbb{Z}_2$ gauge theory

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We study the role of large clusters of center vortices in producing confinement in 3D $\mathbb{Z}_2$ gauge theory. First, we modify each configuration of a Monte Carlo–generated ensemble in the confined phase by removing the largest cluster of center vortices, and show that the ensemble thus obtained does not confine. Conversely, we show that by removing all of the small clusters of center vortices and leaving the largest one only, confinement is preserved, albeit with a string tension significantly smaller than the original one. Remarkably, also the string corrections due to the quantum fluctuations of the confining flux tube are preserved by this transformation.

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It is widely believed that center vortices play an important role in producing confinement, by disordering the gauge configurations and hence making the Wilson loop decay with the area law. The idea itself is quite old [1,2], but its investigation by lattice methods was initiated a few years ago in Ref. [3]. A very recent review can be found in [4].

3D $\mathbb{Z}_2$ gauge theory is the simplest nontrivial gauge theory to display confinement: due to the moderate size of its configuration space, it lends itself to very high precision Monte Carlo simulations. Moreover the gauge group coincides with its center, so that the identification of center vortices can be performed without resorting to any gauge-fixing procedure. Therefore $\mathbb{Z}_2$ gauge theory appears as an ideal laboratory to test the actual relevance of center vortices to the mechanism of confinement.

On the other hand, the role of center vortices in confinement in this model is in a way trivial, since removal of *all* center vortices from $\mathbb{Z}_2$ gauge theory configurations simply removes all the dynamics by transforming every configuration into the trivial vacuum. Therefore center vortices are, in this sense, trivially responsible not only for confinement, but for the whole dynamics of the model.

In this study we investigate a subtler issue, namely the effect of the *size* of clusters of center vortices on confinement. The hypothesis we want to test, first proposed in Ref. [5], is that confinement in this model is due to the existence of an infinite cluster of center vortices, that is, a connected component, in the graph defined in the dual lattice by all the center vortices, whose size scales linearly with the lattice volume. Only such a giant component in the graph is able to disorder the gauge configurations enough to produce the area-law decay of the Wilson loop.[1]

The $\mathbb{Z}_2$ gauge model is defined by the action

$$S(\beta)=-\beta\sum_{\square}\sigma_{\square}, \quad \sigma_{\square}=\prod_{l\in\square}\sigma_l \tag{1}$$

where the sum is extended to all plaquettes of a cubic lattice, on whose links l $\mathbb{Z}_2$ variables σ are defined: each plaquette contributes the product of its links to the action. Center vortices are constructed by assigning a vortex in the dual lattice to each frustrated plaquette in the direct lattice. Since, in the direct lattice, the product of the six plaquettes forming a cube is constrained to be equal to 1, the resulting graph of center vortices in the dual graph has an even coordination number.

Such a graph is in general made of many connected components, that we call *clusters* of center vortices. The value of a Wilson loop in a given configuration is ±1 according to the number, modulo 2, of frustrated plaquettes of an arbitrary surface bounded by the loop, or, in the center vortex language, to the number, modulo 2, of vortex lines that are linked to the loop.

A plausible argument for the crucial role of the infinite cluster of vortex lines goes as follows (see also Ref. [5]): first, note that the only clusters contributing to the Wilson loop $W(C)$ are those linked to C, as the property of even coordination number requires. Assume now that the clusters of center vortices have a maximum size of order R: the contribution to a Wilson loop of much larger size comes only from the clusters located near the loop C. The number of these clusters grows linearly with the length of the loop and produces the decay of $\langle W(C)\rangle$ with the perimeter law and the theory is deconfined. If such a maximum size R does not

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[1]In the 3D $\mathbb{Z}_2$ gauge theory we are studying there exists also a *different* kind of cluster whose percolation is associated to confinement: the Fortuin-Kasteleyn clusters defined in the dual spin model [6]. See also Ref. [7] for an analysis of the relationships between these and other condensates in the case of $\mathbb{Z}_2$ gauge theory with dynamical matter.

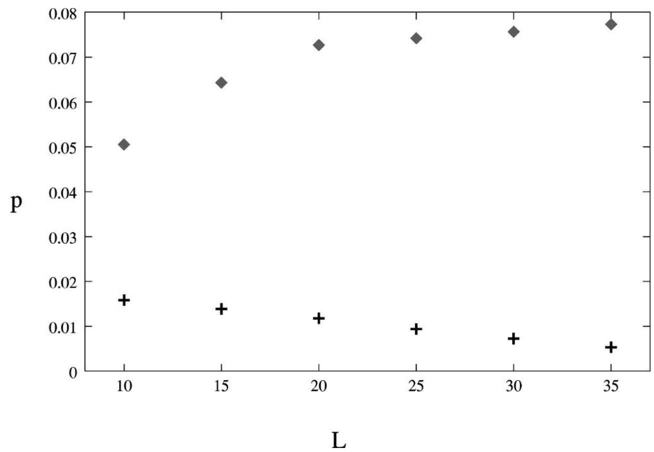

FIG. 1. The diamonds represent the density p of dual links belonging to the largest cluster, as a function of the lattice size: the fact that such density tends to a constant means that the largest cluster of center vortices is infinite in the thermodynamic limit. The same density for the second largest cluster tends to zero (crosses).

exist, that is if there is an infinite cluster, then the number of vortex lines linked to the loop grows with its area, and we have confinement.

It is straightforward to verify numerically that in the confining phase of a $\mathbb{Z}_2$ gauge system there is no ambiguity in finding a cluster of center vortices whose size scales linearly with the lattice volume for large enough lattices (see Fig. 1), while in the deconfined phase we found that the density of the largest cluster decreases rapidly with the volume.

It has to be noted that the presence of the infinite cluster does not necessarily imply a percolation property of the central vortices. Confinement requires merely the presence of an infinite cluster, while percolation demands an infinite cluster of large enough density.

To study more carefully the relationship between the presence of the infinite cluster and the value string tension, we chose to simulate the model at $\beta=0.74883$, which is well inside the scaling region, and for which the value of the string tension is known with high precision from simulations of the dual model, that is, the 3D (spin) Ising model [8,9]

$$\sigma=0.01473(10). \tag{2}$$

First, we verified the *existence* of an ‘‘infinite’’ cluster of vortex lines: for each configuration, we selected the largest connected component of the graph defined by the center vortices, and verified that the size of such a component grows linearly with the volume of the lattice.

The results of this analysis are shown in Fig. 1, where the size of the largest cluster divided by the lattice volume is shown to approach a constant for large lattices. The size of the second largest cluster is also shown: its size relative to the lattice volume tends to zero and the identification of the ‘‘infinite’’ cluster is unambiguous. To test the relevance of the largest cluster of center vortices, we proceeded as follows: first, we modified each configuration in the Monte Carlo ensemble by eliminating all the vortices not belonging to the largest cluster, and second, by eliminating, instead, the

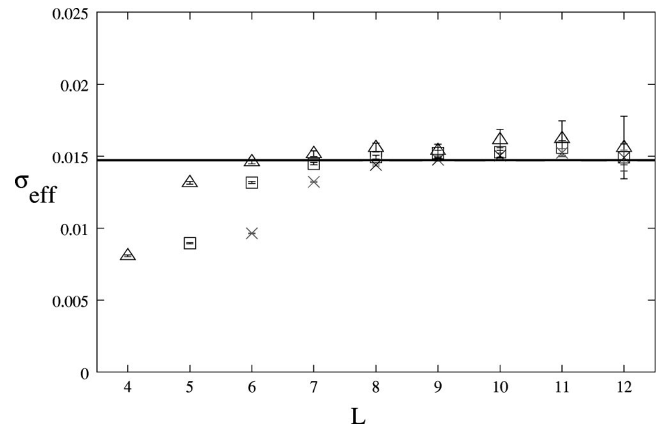

FIG. 2. The estimator $\sigma_{\rm eff}$ of the string tension for $n=2$ (triangles), 3 (squares), and 4 (crosses), as a function of L. The horizontal line is the string tension $\sigma=0.1473$, taken from the literature.

largest cluster only. The qualitative picture described above suggests that a non-zero string tension will be found in the first case but not in the second.

An efficient method to extract the string tension from Wilson loop data generated by Monte Carlo simulations, which takes into account the string fluctuation contribution, was introduced in Ref. [10]. One defines the ratio of the expectation values of rectangular Wilson loops with the same perimeter:

$$r(L,n)=\frac{\langle W(L+n,L-n)\rangle}{\langle W(L,L)\rangle}. \tag{3}$$

If a simple area law described the Wilson loop behavior, such ratios would behave for large L as

$$r(L,n)\sim\exp(\sigma n^2). \tag{4}$$

The quantum fluctuations of the color flux tube induce a correction to this behavior that depends on the ratio n/L only:

$$r(L,n)\sim\exp(\sigma n^2)F(n/L) \tag{5}$$

where

$$F(t)=\left[\frac{\eta(i)\sqrt{1-t}}{\eta\left(i\,\dfrac{1+t}{1-t}\right)}\right]^{1/2} \tag{6}$$

and η is the Dedekind function, so that one can define the modified ratios

$$s(L,n)=\frac{r(L,n)}{F(n/L)} \tag{7}$$

which in turn can be used to evaluate the string tension, since the quantity

$$\sigma_{\rm eff}(L,n)\equiv\frac{\log s(L,n)}{n^2} \tag{8}$$

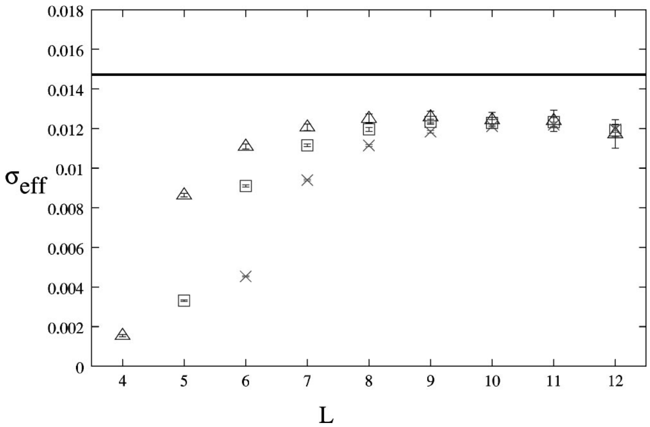

FIG. 3. Same as Fig. 2 for the configurations where all of the vortices except the ones belonging to the largest cluster have been removed.

approaches the string tension for large L.

A test of the correctness of the description Eq. (6) of the string fluctuation is obtained by verifying that the estimator $\sigma_{\rm eff}$ defined in Eq. (8) reaches a constant value for smaller values of L than the simple estimator which neglects string fluctuations:

$$\sigma_{\text{no string}} \equiv \frac{\log r(L,n)}{n^2}. \tag{9}$$

In Fig. 2 we display the behavior of $\sigma_{\rm eff}(L,n)$ as a function of L for $n=2,3,4$ for the original configurations. Figure 3 shows the same for the configurations where only the largest cluster of center vortices has been left, while Fig. 4 is obtained by removing only the largest cluster. These figures show that our main expectations are fulfilled: by removing the finite clusters one still obtains a confining theory (Fig. 3), while when removing the ''infinite'' cluster the string tension vanishes (Fig. 4).

However by examining Fig. 3 one can learn two nontrivial lessons: first, the string tension of the ensemble where only the largest cluster has been retained is definitely smaller than the one of the full model: while it is true (as shown by Fig. 4) that finite clusters cannot by themselves induce confinement, it is not true that they do not contribute to the string tension.[2]

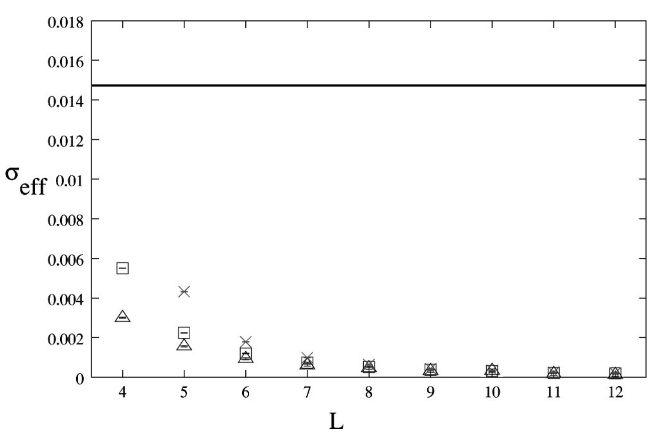

FIG. 4. Same as Fig. 2 for the configurations where the vortices belonging to the largest cluster have been removed.

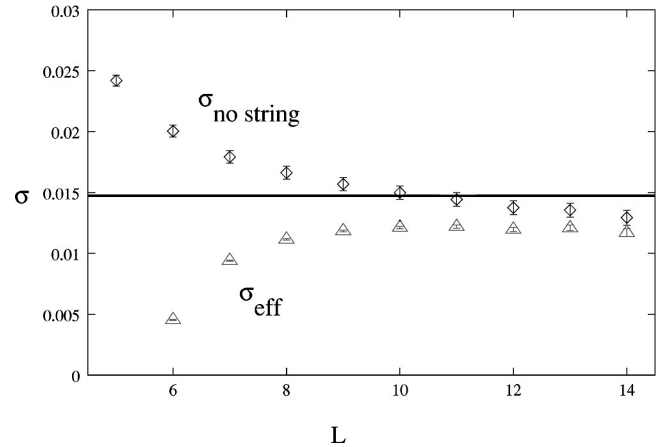

FIG. 5. Comparison between the two estimators $\sigma_{eff}(L,n)$ and $\sigma_{\text{no string}}(L,n)$. The data refer to $n=4$ and to the configurations where all but the largest cluster have been removed.

Second, Fig. 3 shows that the corrections due to the flux tube fluctuations survive the elimination of the small clusters: indeed Fig. 3 shows the estimator $\sigma_{\rm eff}$ defined in Eq. (8) which includes the string fluctuation contribution. The quantitative importance of such contributions is shown by comparing Fig. 3 to Fig. 5, where the two string tension estimators $\sigma_{\rm eff}$ and $\sigma_{\text{no string}}$ are compared for the $n=4$ data in the configurations in which only the largest cluster has been left.

In conclusion, our results confirm the picture of confinement as due to the existence of an infinite cluster of center vortices: our choice of the $\mathbb{Z}_2$ gauge theory allows us to bypass all the problems related to the gauge-fixing and center projection that one encounters when studying the same issue in $SU(N)$ gauge theories.

Two important new facts emerge from our study.

(i) While the largest center vortex is responsible for confinement, since its removal from the configurations makes the string tension vanish, the string tension measured from configurations in which all the other clusters have been removed does not reproduce the full string tension of the original theory. Therefore small clusters of vortices, while unable by themselves to disorder the system enough to produce confinement, do give a finite contribution to the string tension of the full theory.

(ii) The quantum fluctuations of the flux tube survive the elimination of the small clusters: the Wilson loop after deletion of all the small clusters shows the same shape dependence as the ones of the full theory, which can be explained as originating by the fluctuations of a free bosonic string.

[2]Note that the qualitative argument given above would imply that the string tension generated by the largest cluster should equal the full one *only* by assuming that the space-time distributions of the infinite and finite clusters are uncorrelated, an assumption which is obviously unjustified in any non-trivial theory.

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